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Vacuum Energies and Effective Potential in Light-Cone Field Theories

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Abstract

Vacuum energies are computed in light-cone field theories to obtain effective potentials which determine vacuum condensate. Quantization surfaces interpolating between the light-like surface and the usual spatial one are useful to define the vacuum energies unambiguously. The Gross-Neveu, $SU(N)$ Thirring, and $O(N)$ vector models are worked out in the large N limit. The vacuum energies are found to be independent of the interpolating angle to define the quantization surface. Renormalization of effective potential is explicitly performed. As an example of the case with nonconstant order parameter, two-dimensional QCD is also studied. Vacuum energies are explicitly obtained in the large N limit which give the gap equation as the stationary point.

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1. Introduction

Quantization on light-cone has been proposed to offer a nonperturbative method for field theories [1], [2]. It is relatively easy to identify genuine dynamical degrees of freedom in this method. One of the basic reason for this simplicity is kinematical: light cone momentum $p_+ = \frac{p_0+p_1}{\sqrt{2}}$ for a particle is always positive. Therefore the particle-antiparticle pair condensation is forbidden by the light-cone momentum conservation alone. Therefore the vacuum in the light-cone limit is apparently the trivial Fock vacuum. By virtue of the trivial vacuum, one can easily compute, for example, mass spectra and wave functions [3]. To derive these quantities more efficiently, discretized light-cone method or light-cone Tamm-Dancoff method have been proposed and have produced interesting results [4], [5].

On the other hand, there are some drawbacks in the light-cone field theories. Firstly, loss of manifest covariance generally complicates the renormalization procedure of light-cone field theories, since counterterms are no longer restricted by the covariance [6]. More importantly, it is difficult to uncover the vacuum structure such as the vacuum condensate or the spontaneous symmetry breaking. The question of vacuum structure is usually analyzed in terms of zero mode constraints [7]. To explain zero mode analysis for spontaneous symmetry breaking, let us consider the scalar ϕ^4 model in two dimensions in light-cone coordinates $x^+ = \frac{x^0+x^1}{\sqrt{2}}$ and $x^- = \frac{x^0-x^1}{\sqrt{2}}$,

$$\mathcal{L} = \partial_+\phi\partial_-\phi - V(\phi), \qquad V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4. \tag{1.1}$$

Defining the canonical momentum π introduces a primary constraint

$$\pi \equiv \frac{\partial\mathcal{L}}{\partial\partial_+\phi} = \partial_-\phi. \tag{1.2}$$

By adding the constraint with an arbitrary coefficient $v(x)$, the Hamiltonian is given by

$$\mathcal{H} = \pi\partial_+\phi - \mathcal{L} + v\cdot(\pi - \partial_-\phi) = V(\phi) + v\cdot(\pi - \partial_-\phi)\,. \tag{1.3}$$

Since the nonzero modes of the primary constraint (1.2) is of second class, the time evolution of the primary constraint determines $v(x)$

$$[(\pi-\partial_-\phi)(x), \mathcal{H}(y)]\,\delta(x^+-y^+) = -i\left(\frac{dV}{d\phi}(x) + 2\partial_- v(x)\right)\delta^{(2)}(x-y) \approx 0. \tag{1.4}$$

However, the zero mode part of the Eq. (1.4) gives the secondary constraint which is called the zero mode constraint

$$0 = \int_{-L}^{L} dx^- \frac{dV}{d\phi}(x) = m^2\int_{-L}^{L} dx^-\phi(x) + \frac{\lambda}{3!}\int_{-L}^{L} dx^-\phi^3(x), \tag{1.5}$$

where we compactify the spatial direction and impose a periodic boundary condition to define the zero mode of ϕ unambiguously. Eq. (1.5) shows that the zero mode is not an independent variable, but is given as a nonlinear expression of nonzero modes. By laboriously analyzing the constraint (1.5), one can find a solution with nonvanishing zero modes in certain cases which indicates the spontaneous symmetry breaking [7]

$$\int_{-L}^{L} dx^- \phi(x) \neq 0. \tag{1.6}$$

One should distinguish two kinds of zero modes. One type is the zero mode associated to the above constraint. The other zero mode is the dynamical zero mode of gauge fields which arises because of the nontrivial topology due to the compactified spatial dimensions [8]. The former is directly related to the question of vacuum condensate or the spontaneous symmetry breaking, whereas the latter is often be responsible to nonperturbative effects associated to the gauge fields . There has also been a number of works aiming at determining vacuum structures with methods like Hartree type equations in the light-cone field theories [9]. It has been proposed to use regularizations to define the light-like quantization surface as a limit of space-like surfaces [9], [10]. On the other hand, the most efficient method to find the vacuum condensate in the covariant approach is usually to compute vacuum energies and to obtain the effective potential [11]–[14]. More complicated models such as the two-dimensional QCD coupled to quarks in the fundamental representation are also studied in the light-cone gauge using the large N limit [15]–[17]. It has been observed that the chiral symmetry breaking occurs in the large N limit and the quark-antiquark condensation has been computed [18]–[20], [9]. Higher order corrections in the $1/N$ expansion [21] convert this spontaneous symmetry breaking to the Berezinski-Kosterliz-Thouless phenomenon [22] and make the result consistent with the Coleman's theorem [23].

The purpose of our paper is to compute vacuum energies explicitly in light-cone field theories and to demonstrate that the effective potential can be obtained to determine nontrivial vacuum condensate. In order to define the vacuum energies unambiguously, a regularization is extremely useful to define the light-like quantization surface as a limit of space-like surfaces. We use space-like quantization surfaces which interpolate between the ordinary spatial surface and the light-like surface [10]. Light-cone quantization is defined as a limit from the space-like surface to the light-like one. This method enabled us to compute the effective potential of light-cone field theories unambiguously. As illustrative examples for the effective potential with constant order parameters, we have studied the Gross-Neveu model, the $SU(N)$ Thirring model in two-dimensions, and the $O(N)$ vector models in two, three, and four dimensions using the large N limit. The previous treatments of these models employed Hartree type methods and did not compute vacuum energies and effective potentials [24]. We find that the vacuum energies are independent of the interpolating angle to define the quantization surface. We have performed

the renormalization of the effective potential explicitly [13]. As an example of the case with nonconstant order parameter, we have also studied the two-dimensional QCD with quarks in the fundamental representation. We explicitly obtain in the large N limit the vacuum energies which give the gap equation as the stationary point. The gap equation turns out to depend on the interpolating angle which defines the quantization surface and the gauge parameter. In the limit of spatial quantization surface, our gap equation agrees with the axial gauge result [20].

Our results suggest that one can neglect the constraint zero mode problem once the possible vacuum condensate is determined by our method of vacuum energy and effective potential. Although the zero mode fluctuations around the vacuum value give induced interactions among nonzero modes through the zero mode constraint, these interaction terms are always multiplied by inverse powers of the length of the compact spatial dimension and should disappear as we let the length to go to infinity. The only subtlety should lie in the determination of the vacuum condensate, and it can be most efficiently incorporated by means of effective potential. Therefore we propose as a practical method that the possible vacuum condensate be determined by using our vacuum energy and effective potential and that the induced interactions due to the zero mode fluctuations should be neglected in using the discretized light-cone or other approaches to obtain mass spectra and wave functions.

In sect. 2, we study the Gross-Neveu model and the $SU(N)$ Thirring model. In sect. 3, the $O(N)$ $\lambda\phi^4$ model is worked out. In sect. 4, we compute vacuum energies of QCD. Our conventions and useful formulas are summarized in appendix.

2. Gross-Neveu Model and Its Generalizations

2.1. Massive Gross-Neveu Model

We consider the large N limit of the Gross-Neveu model which contains a four-fermion interaction among N component Dirac fields $\psi^a, a = 1, \cdots, N$ in two dimensions [12]

$$\mathcal{L} = \bar{\psi}^a \left(i\gamma^\mu \partial_\mu - m_0 \right) \psi^a + \frac{g_0}{2N} \left(\bar{\psi}^a \psi^a \right)^2, \tag{2.1}$$

where m_0 and g_0 are bare mass and bare coupling constant, respectively. This model has a global $U(N)$ symmetry $\psi^a \to U^a{}_b \psi^b$. Moreover, it possesses a discrete chiral symmetry when $m_0 = 0$

$$\psi^a \to \gamma_5 \psi^a. \tag{2.2}$$

By introducing an auxiliary field σ, we can obtain an equivalent Lagrangian

$$\mathcal{L}_\sigma = \bar{\psi}^a \left(i\gamma^\mu \partial_\mu - \sigma\right)\psi^a + \frac{Nm_0}{g_0}\sigma - \frac{N}{2g_0}\sigma^2, \tag{2.3}$$

which reduces to the original one (2.1) after integrating over σ, since

$$\mathcal{L} = \mathcal{L}_\sigma + \frac{N}{2g_0}\left(\sigma + \frac{g_0}{N}\bar{\psi}^a\psi^a - m_0\right)^2 - \frac{Nm_0^2}{2g_0}. \tag{2.4}$$

Our goal is to compute vacuum energies in the light-cone quantization. This procedure, however, encounters ill-defined quantities if one performs quantization naively on light-like surface. In order to overcome this problem, we shall define the light-like quantization surface as a limit from the space-like surface. This procedure can be regarded as a regularization to define the singular light-cone quantization properly. In this way, we can unambiguously compute the vacuum energies. To this end, we use the coordinate system which interpolates the light-cone and ordinary coordinates [10]

$$\begin{pmatrix} x^+ \\ x^- \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}, \tag{2.5}$$

where θ is a parameter defined in the region $\frac{\pi}{2} < \theta \leq \pi$. In this frame, metric tensor becomes

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} c & s \\ s & -c \end{pmatrix}, \quad \mu,\nu = +,-, \quad c \equiv -\cos\theta,\ s \equiv \sin\theta. \tag{2.6}$$

In quantization, we regard x^+ and x^- as time and space, respectively. Note that ordinary time quantization corresponds to the limit

$$\theta \to \pi, c = -\cos\theta \to 1, s = \sin\theta \to 0, \qquad x^+ = x^0, \quad x^- = -x^1, \tag{2.7}$$

and light-cone quantization corresponds to the limit

$$\theta \to \frac{\pi}{2}, c = -\cos\theta \to 0, s = \sin\theta \to 1, \qquad x^+ = \frac{x^0 + x^1}{\sqrt{2}},\ x^- = \frac{x^0 - x^1}{\sqrt{2}}, \tag{2.8}$$

Let us emphasize that this change of quantization surface is nothing to do with the Lorentz transformation. Conjugate momentum for ψ^a is defined by

$$\pi^a(x) = \frac{\partial\mathcal{L}_\sigma}{\partial\partial_+\psi^a(x)} = i\bar{\psi}^a\gamma^+, \tag{2.9}$$

whose components are given explicitly in eq.(A.4) in appendix. We can apply the ordinary quantization of Dirac particle as long as $s \neq 1$ and impose an anticommutation relation at equal time

$$\left\{\psi^a_\alpha(x), \pi^b_\beta(y)\right\}_{x^+=y^+} = i\delta_{\alpha\beta}\delta^{ab}\delta(x^- - y^-). \tag{2.10}$$

Hamiltonian density is

$$\mathcal{H} = \pi^a \partial_+ \psi^a - \mathcal{L}_\sigma = \bar{\psi}^a(-i\gamma^- \partial_- + \sigma)\psi^a - \frac{N m_0}{g_0}\sigma + \frac{N}{2g_0}\sigma^2. \tag{2.11}$$

In the large N limit, vacuum energy is given by the fermion one loop contributions. Therefore we shall treat the auxiliary field σ as a background field. Since we are interested in the effective potential to determine the vacuum expectation value, we take σ as a constant background. There exist only quadratic terms in the quantum field ψ^a in the Lagrangian. By solving the equation of motion for ψ^a, we obtain the light-cone energy

$$p_+ = \frac{1}{c}\left(-sp_- + \omega_p\right), \quad \omega_p = \sqrt{(p_-)^2 + c\sigma^2}. \tag{2.12}$$

and the corresponding spinor is given in eq.(A.5) in appendix. If we take the light-like limit $c = -\cos\theta \to 0, s = \sin\theta \to 1$, we obtain finite energy only for positive momenta $p_- > 0$ as shown in Fig.1.

$$\lim_{c\to 0} p_+ = \frac{\sigma}{2p_-}. \tag{2.13}$$

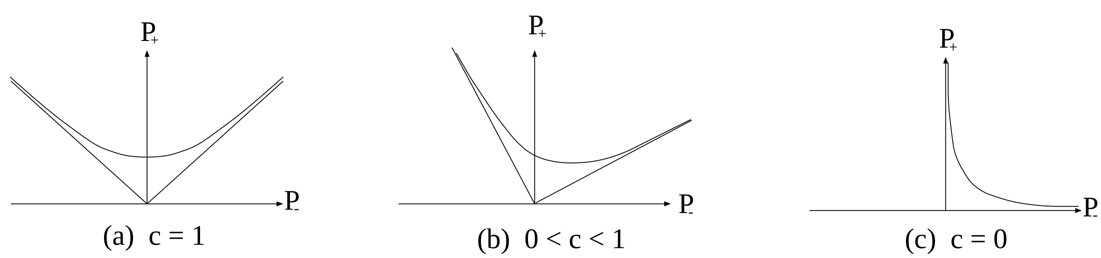

Figure 1: Dispersion relations of a free massive particle on (a) usual, (b) an interpolating and (c) light-cone quantization surfaces.

To avoid possible infrared divergences we compactify x^- direction and impose a periodic boundary condition $\psi^a(x^-) = \psi^a(x^- + 2L)$, which gives discrete momenta

$$p_{n-} = \frac{\pi n}{L}, \quad n \in \mathbf{Z}. \tag{2.14}$$

Other boundary conditions are also allowed. Therefore the fermion field operator is expanded into modes

$$\psi^a(x) = \frac{1}{\sqrt{2L}}\sum_n \left[\mathrm{e}^{-ip_n x} u(p_n) b_n^a + \mathrm{e}^{ip_n x} v(p_n) d_n^{a\dagger}\right], \tag{2.15}$$

where $p_n = (p_{n+}, p_{n-})$ is defined in eq. (2.12) and (2.14). Anticommutation relation (2.10) becomes

$$\{b_n^a, b_m^{b\dagger}\} = \delta_{n,m}\delta^{ab}, \quad \{d_n^a, d_m^{b\dagger}\} = \delta_{n,m}\delta^{ab}. \tag{2.16}$$

The Hamiltonian is given by using (2.11) and (2.15)

$$P_+ = \int_{-L}^{L} dx^- \mathcal{H}(x) = \sum_n p_{n+}\left(b_n^{a\dagger}b_n^a - d_n^a d_n^{a\dagger}\right) + 2L\left(-\frac{Nm_0}{g_0}\sigma + \frac{N}{2g_0}\sigma^2\right). \tag{2.17}$$

Since the vacuum of this Hamiltonian is the Fock vacuum satisfying $b_n^a |0\rangle = d_n^a |0\rangle = 0$, we obtain the vacuum energy density in the leading order of the $1/N$ expansion

$$\frac{1}{2L}\langle 0| P_+ |0\rangle = V(\sigma), \tag{2.18}$$

$$V(\sigma) = V_0 - \frac{Nm_0}{g_0}\sigma + \frac{N}{2g_0}\sigma^2 + V_{1-\mathrm{loop}}(\sigma), \tag{2.19}$$

$$V_{1-\mathrm{loop}}(\sigma) = -\frac{N}{2L}\sum_n p_{n+} = -\frac{N}{2L}\sum_n \frac{1}{c}\left[-s\frac{n\pi}{L} + \sqrt{\left(\frac{n\pi}{L}\right)^2 + c\sigma^2}\right]. \tag{2.20}$$

We have introduced a constant V_0 to renormalize the cosmological constant. We observe that $V_{1-\mathrm{loop}}$ appears to depend on the parameter $c = -\cos\theta, s = \sin\theta$ in eq.(2.5) to define the quantization surface. Since one-loop vacuum energy density has no infrared divergence, we can now take $L \to \infty$ limit and obtain

$$V_{1-\mathrm{loop}}(\sigma) = -N\int \frac{dp_-}{2\pi}\left[\frac{-sp_- + \sqrt{(p_-)^2 + c\sigma^2}}{c}\right]. \tag{2.21}$$

Since the vacuum energy density is UV divergent, we apply the Pauli-Villars regularization

$$V_{1-\mathrm{loop}}^{PV}(\sigma) = \lim_{\Lambda_i\to\infty}\left[V_{1-\mathrm{loop}}(\sigma) - \sum_i a_i V_{1-\mathrm{loop}}(\Lambda_i)\right]. \tag{2.22}$$

By requiring $\sum_i a_i = 1, \quad \sum_i a_i\Lambda_i^2 = \sigma^2$, we obtain

$$V_{1-\mathrm{loop}}^{PV}(\sigma) = -\frac{N}{4\pi}\lim_{\Lambda_i\to\infty}\left(\sum_i a_i\Lambda_i^2\log\Lambda_i^2 - \sigma^2\log\sigma^2\right). \tag{2.23}$$

We find that the positive ($p_- > 0$) and negative ($p_- < 0$) momentum region contribute equally to the one-loop effective potential. It is interesting to observe that the vacuum energy no longer depends on the parameter $c = -\cos\theta, s = \sin\theta$ in eq.(2.23) in contrast to the expression (2.21) before the UV regularization and the momentum integration. The result is identical to that

obtained by the usual covariant calculations. On the other hand, if we take the limit of light-like quantization surface $c = -\cos\theta \to 0$ before integrating over the momentum p_-, we find that only the positive momentum region contributes as seen from eq.(2.13). The integral becomes infrared divergent. Even if we regularize the integral by introducing the compact spatial dimension, the contribution of zero mode is still ambiguous and the nonzero mode alone gives a result different from the above. As Fig.1 suggests, this situation arises since contributions from $p_- < 0$ region are squeezed into the infrared divergent zero mode contribution and become ambiguous if the light-cone limit is taken inside the momentum integral. Therefore we conclude that the light-cone limit does not commute with the momentum integration and that the light-cone limit must be taken after integrating over the momentum. To test the sensitivity of the procedure to define the light-like surface as a limit of the spacelike surface, we have also computed the vacuum energy using another choice of the limit from the spacelike surface [9] with $x^+ = \frac{1}{\sqrt{2}}\left[\left(1+\frac{\epsilon}{2L}\right)x^0 + \left(1-\frac{\epsilon}{2L}\right)x^1\right]$ and $x^- = \frac{1}{\sqrt{2}}\left[x^0 - x^1\right]$, and have found the identical result provided we perform momentum integration before taking the limit.

A simple choice of the Pauli-Villars regulators $a_1 = 2, \quad a_2 = -1, \quad \Lambda_2^2 = 2\Lambda_1^2 - \sigma^2$ gives

$$V_{1-\mathrm{loop}}^{PV}(\sigma) = -\frac{N}{4\pi}\lim_{\Lambda_1\to\infty}\left[\sigma^2\left(\log\frac{2\Lambda_1^2}{2\sigma^2}+1\right) - 2\Lambda_1^2\log 2\right]. \tag{2.24}$$

We impose renormalization conditions for the cosmological constant, the mass, and the coupling constant

$$V(\sigma=0)=0, \quad \frac{\partial V}{\partial\sigma}(\sigma=0) = -\frac{Nm}{g}, \quad \frac{\partial^2 V}{\partial\sigma^2}(\sigma=\mu) = \frac{N}{g}, \tag{2.25}$$

where m and g are the renormalized mass and the renormalized coupling constant, respectively. The renormalized effective potential is finally given by

$$V(\sigma) = -\frac{Nm}{g}\sigma + \frac{N}{2g}\sigma^2 + \frac{N}{4\pi}\left[\sigma^2\left(\log\frac{\sigma^2}{\mu^2}+1\right)\right]. \tag{2.26}$$

Minimizing the effective potential, we obtain a nonvanishing vacuum expectation value for σ

$$\sigma \approx \frac{m}{|m|}\mu e^{-1-\frac{\pi}{g}} + \frac{\pi m}{g}, \tag{2.27}$$

which implies the spontaneous breakdown of the discrete chiral symmetry (2.2). As was discussed in [12], it follows that the chiral condensate occurs $\langle\bar{\psi}^a\psi^a\rangle \neq 0$.

2.2. $SU(N)$ Thirring Model

The $SU(N)$ Thirring model is a generalization of the Gross-Neveu model [21]

$$\mathcal{L} = \bar{\psi}^a \left(i\gamma^\mu \partial_\mu - m_0\right) \psi^a + \frac{g_0}{2N} \left[\left(\bar{\psi}^a \psi^a\right)^2 - \left(\bar{\psi}^a \gamma_5 \psi^a\right)^2 \right]. \tag{2.28}$$

Using auxiliary fields σ and π corresponding to the scalar and pseudoscalar fermion bilinears, we obtain the equivalent Lagrangian

$$\mathcal{L}_{\pi\sigma} = \bar{\psi}^a \left(i\gamma^\mu \partial_\mu - \sigma - i\pi\gamma_5\right) \psi^a + \frac{N m_0}{g_0}\sigma - \frac{N}{2g_0}\left(\sigma^2 + \pi^2\right). \tag{2.29}$$

This model has a global $U(N)$ symmetry $\psi^a \to U^a{}_b \psi^b$. It also has a continuous chiral symmetry when $m_0 = 0$

$$\psi^a \to \mathrm{e}^{i\beta\gamma_5}\psi^a, \quad \sigma + i\pi \to \mathrm{e}^{-2i\beta}\left(\sigma + i\pi\right). \tag{2.30}$$

Following the same procedure as that of the Gross-Neveu model, we obtain the renormalized effective potential in the large N limit

$$V(\sigma, \pi) = -\frac{Nm}{g}\sigma + \frac{N}{2g}\left(\sigma^2 + \pi^2\right) + \frac{N}{4\pi}\left[\left(\sigma^2 + \pi^2\right)\left(\log\frac{\sigma^2 + \pi^2}{\mu^2} + 1\right)\right], \tag{2.31}$$

which depends on two spacetime-independent background fields σ and π.

By minimizing this effective potential, one finds that the σ acquires a nonvanishing vacuum expectation value and the continuous chiral symmetry is spontaneously broken in two dimensions. It has been observed [21] that higher order contributions introduces power law decay for the correlation function $\langle \bar{\psi}^a(1-\gamma^5)\psi^a(x)\bar{\psi}^b(1+\gamma^5)\psi^b(y)\rangle \propto |x-y|^{-\frac{1}{N}}$ as $|x-y| \to \infty$. This Berezinski-Kosterliz-Thouless type behavior [22] makes the correlation function compatible with the Coleman's theorem [23]. Since this behavior shows that the chiral symmetry is almost broken, the leading order result gives physically correct picture of the Gross-Neveu model.

3. $O(N)$ $\lambda\phi^4$ Model in d-Dimensions

In this section, we consider N component scalar field ϕ^a ($a = 1, \cdots, N$) with the $O(N)$ invariant quartic interaction in d-dimensions ($d = 2, 3, 4$)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - \frac{m_0^2}{2}\phi^a\phi^a - \frac{g_0}{8N}\left(\phi^a\phi^a\right)^2. \tag{3.1}$$

This theory is invariant under global $O(N)$ transformations $\phi^a \to U^a{}_b\phi^b$. We shall show that the vacuum energy of this model can be defined in the light-cone quantization. Introducing an auxiliary field σ, we obtain an equivalent Lagrangian

$$\mathcal{L}_\sigma = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\sigma\phi^a\phi^a - \frac{Nm_0^2}{g_0}\sigma + \frac{N}{2g_0}\sigma^2. \tag{3.2}$$

In the interpolating coordinates (2.5), the Lagrangian becomes

$$\mathcal{L}_\sigma = \frac{c}{2}\left[(\partial_+\phi^a)^2 - \left(\partial^-\phi^a\right)^2\right] + s\partial_+\phi^a\partial_-\phi^a - \frac{1}{2}(\partial_\perp\phi^a)^2 - \frac{1}{2}\sigma\phi^a\phi^a - \frac{Nm_0^2}{g_0}\sigma + \frac{N}{2g_0}\sigma^2. \tag{3.3}$$

Conjugate momentum for ϕ^a is defined by

$$\pi^a = \frac{\partial\mathcal{L}_\sigma}{\partial\partial_+\phi^a} = c\partial_+\phi^a + s\partial_-\phi^a. \tag{3.4}$$

Regarding the auxiliary field σ as spacetime-independent background field as before, this model reduces to free massive bosons. To avoid possible infrared divergences, we compactify x^-,$x^\perp$ directions and impose periodic boundary conditions $\phi^a(x^-,x^\perp) = \phi^a(x^-+2L,x^\perp) = \phi^a(x^-,x^\perp+2L)$, where $x^\perp = x^i, \quad i = 2,3,\cdots,d-1$. As a result, momenta take discrete values $p_- = \frac{\pi n}{L}, \quad p_\perp = \frac{\pi n}{L}$. Solving the equation of motion, ϕ^a can be expanded into modes

$$\phi^a(x) = \frac{1}{\sqrt{2L}}\sum_n \frac{1}{\sqrt{2\omega_n}}\left[\mathrm{e}^{-ip\cdot x}a^a(n) + \mathrm{e}^{ip\cdot x}a^{a\dagger}(n)\right], \tag{3.5}$$

where $p\cdot x = p_+x^+ + p_-x^- + p_\perp x^\perp$ with $p_+ = (-sp_- + \omega_p)/c, \omega_p = \sqrt{(p_-)^2 + (p^\perp)^2 + c\sigma}$. Imposing the canonical commutation relation at $x^+ = y^+$, we obtain

$$\left[a^a(n), a^{a\dagger}(m)\right] = \delta_{n,m}\delta^{ab}. \tag{3.6}$$

Using (3.5), the Hamiltonian is given by

$$\begin{aligned} P_+ &= \int_{-L}^{L} d^{d-1}x\left[\pi^a\partial_+\phi^a - \mathcal{L}_\sigma\right] \\ &= \sum_n \frac{1}{2}\left[a^{a\dagger}(n)a^a(n) + a^a(n)a^{a\dagger}(n)\right] + (2L)^{d-1}\left(\frac{Nm_0^2}{g_0}\sigma - \frac{N}{2g_0}\sigma^2\right). \end{aligned} \tag{3.7}$$

In the large N limit, vacuum is the Fock vacuum $|0\rangle$ and the vacuum energy density is given by

$$V(\sigma) = \frac{1}{(2L)^{d-1}}\langle 0|P_+|0\rangle, \tag{3.8}$$

$$V(\sigma) = V_0 + \frac{Nm_0^2}{g_0}\sigma - \frac{N}{2g_0}\sigma^2 + V_{1-\mathrm{loop}}(\sigma), \tag{3.9}$$

$$V_{1-\mathrm{loop}}(\sigma) = \frac{N}{(2L)^{d-1}}\frac{1}{2}\sum_n \left[\frac{-sp_- + \sqrt{(p_-)^2 + c((p^\perp)^2+\sigma)}}{c}\right]. \tag{3.10}$$

Since there is no infrared singularity in the vacuum energy density, we can take $L \to \infty$ by replacing the discrete sum in (3.10) by a momentum integration. Similarly to the case of the Gross-Neveu model, the above expression for the one-loop vacuum energy appears to depend on the parameter $c = -\cos\theta, s = \sin\theta$ in eq.(2.5) to define the interpolating quantization surface. In the following, however, we shall work out explicit forms of the effective potential in the case of $d = 2, 3, 4$, and shall find the result to be independent of the parameter $c = -\cos\theta, s = \sin\theta$ and to agree with those given by the covariant formalism.

In order to define the effective potential as a function of the constant classical field corresponding to the $O(N)$ vector field ϕ^a, we introduce the spacetime-independent source J^a coupled to ϕ^a

$$\mathcal{L}_{\sigma J^a} = \mathcal{L}_\sigma + J^a\phi^a = -\frac{\sigma}{2}\left(\phi^a - \frac{J^a}{\sigma}\right)^2 - \frac{Nm_0^2}{g_0}\sigma + \frac{N}{2g_0}\sigma^2 + \frac{J^aJ^a}{2\sigma}. \tag{3.11}$$

The generating function $W[J^a, \sigma]$ and the classical field φ^a are defined by

$$W(J^a,\sigma) \equiv -\frac{1}{(2L)^{d-1}}\langle 0|\, P_+\, |0\rangle = -\left(\frac{Nm_0^2}{g_0}\sigma - \frac{N}{2g_0}\sigma^2 + V_{1-\mathrm{loop}}(\sigma)\right) + \frac{J^aJ^a}{2\sigma}, \tag{3.12}$$

$$\varphi^a \equiv \frac{\partial W}{\partial J^a} = \frac{J^a}{\sigma}. \tag{3.13}$$

Performing the Legendre transformation, we obtain the effective potential

$$V(\varphi^a,\sigma) = J^a\varphi^a - W(J^a,\sigma) = \frac{1}{2}\sigma\varphi^a\varphi^a + \frac{Nm_0^2}{g_0}\sigma - \frac{N}{2g_0}\sigma^2 + V_{1-\mathrm{loop}}(\sigma), \tag{3.14}$$

which depends on the classical field φ^a and the background field σ.

As can be seen easily, the effective potential is UV divergent and the degree of divergence depends on the dimensions of spacetime. To regularize the UV divergence, we use the Pauli-Villars regularization method

$$V^{PV}_{1-\mathrm{loop}}(\sigma) = \lim_{\Lambda_i\to\infty}\left[V_{1-\mathrm{loop}}(\sigma) - \sum_i a_i V_{1-\mathrm{loop}}(\Lambda_i)\right]. \tag{3.15}$$

We need to renormalize the model in each dimensions separately. In two dimensions, it is easy to see that the one loop contribution to the effective potential is equivalent to that of the

Gross-Neveu model up to a constant factor

$$V^{O(N)}_{1-loop}(\sigma) = -\frac{1}{2} V^{Gross-Neveu}_{1-loop}(\sqrt{\sigma}). \tag{3.16}$$

We need to renormalize the cosmological constant and the mass but not the coupling constant

$$V(\sigma = 0) = 0, \qquad \frac{\partial V}{\partial \sigma}(\sigma = \mu^2) = \frac{N(m^2 - \mu^2)}{g}. \tag{3.17}$$

The renormalized effective potential in two dimensions is given by

$$V(\varphi^a, \sigma) = \frac{1}{2}\sigma\varphi^a\varphi^a + \frac{Nm^2}{g}\sigma - \frac{N}{2g}\sigma^2 + \frac{N}{8\pi}\sigma\left(\log\frac{\mu^2}{\sigma} + 1\right). \tag{3.18}$$

Searching for the stationary point with respect to σ

$$0 = \frac{\partial V}{\partial \sigma}(\varphi^a, \sigma) = \frac{1}{2}\varphi^a\varphi^a + \frac{Nm^2}{g} - \frac{N}{g}\sigma + \frac{N}{8\pi}\log\frac{\mu^2}{\sigma}, \tag{3.19}$$

the effective potential depending only on φ^a is given by

$$V(\varphi^a) \equiv V(\varphi^a, \sigma = \sigma(\varphi^a)) = \frac{N}{2g}\left[\sigma(\varphi^a)\right]^2. \tag{3.20}$$

One finds that φ^a vanishes at the minimum and the $O(N)$ symmetry is not broken. This result is consistent with the Coleman's theorem and the leading order approximation of the $1/N$ expansion yields a reliable result [13].

In three dimensions, the one loop contribution after choosing the Pauli-Villars regulators is

$$V^{PV}_{1-loop}(\sigma) = -\frac{N}{12\pi}\left(\sigma^{\frac{3}{2}} - \sum_i a_i\Lambda_i^3\right). \tag{3.21}$$

We need to renormalize the cosmological constant and the mass but not the coupling constant

$$V(\sigma = 0) = 0, \qquad \frac{\partial V}{\partial \sigma}(\sigma = 0) = \frac{Nm^2}{g}. \tag{3.22}$$

The renormalized effective potential is thus given by

$$V(\varphi^a, \sigma) = \frac{1}{2}\sigma\varphi^a\varphi^a + \frac{Nm^2}{g}\sigma - \frac{N}{2g}\sigma^2 - \frac{N}{12\pi}\sigma^{\frac{3}{2}}. \tag{3.23}$$

Expressing σ by solving the stationarity condition $\partial V/\partial\sigma = 0$, we obtain the effective potential $V(\varphi^a) \equiv V(\varphi^a, \sigma = \sigma(\varphi^a))$ depending only on the classical field φ^a [13].

In four dimensions, the regularized effective potential is given by

$$V^{PV}_{1-loop}(\sigma) = \frac{N}{32\pi}\left(\sigma^2 \log \sigma - \sum_i a_i \Lambda_i^4 \log \Lambda_i^2\right). \tag{3.24}$$

In contrast to two and three dimensions, the effective potential in four dimensions requires renormalization of the cosmological constant, mass and the coupling constant

$$V(\sigma = 0) = 0, \quad \frac{\partial V}{\partial \sigma}(\sigma = 0) = \frac{Nm^2}{g}, \quad \frac{\partial^2 V}{\partial \sigma^2}(\sigma = \mu^2) = -\frac{N}{g}. \tag{3.25}$$

These conditions give the renormalized effective potential

$$V(\varphi^a, \sigma) = \frac{1}{2}\sigma\varphi^a\varphi^a + \frac{Nm^2}{g}\sigma - \frac{N}{2g}\sigma^2 + \frac{N}{32\pi}\sigma^2\left[\log\frac{\sigma}{\mu^2} - \frac{3}{2}\right]. \tag{3.26}$$

The background field σ is determined as a function of the classical field φ^a by the stationary condition $\partial V/\partial\sigma = 0$. Eliminating σ one finally obtains the effective potential $V(\varphi^a) \equiv V(\varphi^a, \sigma = \sigma(\varphi^a))$ whose physical meaning is discussed in detail in [13], [14].

4. QCD in Two Dimensions in the Large N Limit

QCD in two dimensions is an another interesting model which exhibits the nontrivial vacuum structure, namely the quark-antiquark condensation in the large N limit [18]–[20]. Similarly to the $SU(N)$ Thirring model, higher order corrections in $1/N$ expansion should introduce the power law decay for the correlation function of $< \bar{\psi}\psi(x)\ \bar{\psi}\psi(y) >\sim |x-y|^{-\frac{1}{N}}$ as $|x-y| \to \infty$, in conformity with the Coleman's theorem [21]. Since power law decay is much milder than the usual exponential decay, the spontaneous breaking is almost realized and the leading order in the $1/N$ expansion gives physically sensible result.

The Lagrangian consists of $SU(N)$ gauge fields A^a_μ and the quark ψ^i in the fundamental representation

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, \tag{4.1}$$

where the field strength $F_{\mu\nu}$ and the covariant derivative D_μ are defined as

$$D_\mu = \partial_\mu + igA_\mu, \qquad A_\mu = A^a_\mu T^a,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu], \qquad \mathrm{tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab}.$$

We adopt the light-cone gauge

$$A_- = 0. \tag{4.2}$$

As an advantage of the light-cone gauge, the remaining gauge field becomes a dependent variable. We can eliminate A_+^a by the Gauss law constraint to obtain the Lagrangian

$$A_+^a = -\frac{g}{\partial_-^2}\bar{\psi}\gamma^+ T^a \psi = -\frac{g}{\partial_-^2} J^{+a}, \tag{4.3}$$

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + \frac{1}{2} J^{+a} \frac{1}{\partial_-^2} J^{+a}. \tag{4.4}$$

Thus, the Hamiltonian density with x^+ as time is given by

$$\mathcal{H} = \bar{\psi}^a(-i\gamma^- \partial_- + m)\psi^a - \frac{1}{2} J^{+a} \frac{1}{\partial_-^2} J^{+a}. \tag{4.5}$$

To find the vacuum state, let us minimizing the expectation value of the Hamiltonian over a trial vacuum state. To exhibit a quark-antiquark condensation, we choose the trial vacuum state $|\Phi\rangle$ which is obtained by the Bogoliubov transformation from the Fock vacuum state $|0\rangle$ [25]

$$|\Phi\rangle = \frac{1}{\sqrt{1+\Phi^2(p_-)}} \Pi_{p_-} \left[1 - \Phi(p_-) b^{i\dagger}(p_-) d^{i\dagger}(-p_-)\right] |0\rangle\,, \tag{4.6}$$

where $b^{i\dagger}(p_-)$ and $d^{i\dagger}(p_-)$ are creation operators for quark and antiquark with momentum p_-, color index i. The state is annihilated by operators given by the Bogoliubov transformation

$$B^i(p_-)\,|\Phi\rangle = D^i(p_-)\,|\Phi\rangle = 0, \tag{4.7}$$

$$\begin{aligned} B^i(p_-) &= \frac{1}{\sqrt{1+\Phi^2(p_-)}} \left[b^i(p_-) + \Phi(p_-) d^{i\dagger}(-p_-)\right], \\ D^{i\dagger}(-p_-) &= \frac{1}{\sqrt{1+\Phi^2(p_-)}} \left[d^{i\dagger}(-p_-) - \Phi(p_-) b^i(p_-)\right]. \end{aligned} \tag{4.8}$$

The quark field can be expressed in terms of the original and transformed operators

$$\begin{aligned} \psi^i(x^-) &= \int \frac{dp_-}{\sqrt{2\pi}} \mathrm{e}^{-ip_- x^-} \left[u(p_-) b^i(p_-) + v(-p_-) d^{i\dagger}(-p_-)\right] \\ &= \int \frac{dp_-}{\sqrt{2\pi}} \mathrm{e}^{-ip_- x^-} \left[U(p_-) B^i(p_-) + V(-p_-) D^{i\dagger}(-p_-)\right], \end{aligned} \tag{4.9}$$

where $u(p_-)$ and $v(p_-)$ are original free massive spinors in eq.(A.5), while $U(p_-)$ and $V(p_-)$ are transformed ones in eq.(A.8) in appendix. The commutator of quark fields at equal time can be parametrized by the order parameter $\Phi(p_-)$ of quark-antiquark condensation

$$\begin{aligned}
&\langle\Phi| \frac{1}{2}\left[\psi^i_\alpha(0,x^-), \bar{\psi}^{\beta j}(0,y^-)\right]|\Phi\rangle \\
= \ &\frac{1}{2}\delta^{ij}\int\frac{dp}{2\pi}\mathrm{e}^{-ip(x^--y^-)}\left[\left(\frac{1-\Phi^2(p)}{1+\Phi^2(p)}\frac{m}{\omega_p}-\frac{2\Phi(p)}{1+\Phi^2(p)}\frac{p}{\sqrt{c}\omega_p}\right)\right. \\
&\left.-\left(\frac{1-\Phi^2(p)}{1+\Phi^2(p)}\frac{p}{c\omega_p}+\frac{2\Phi(p)}{1+\Phi^2(p)}\frac{m}{\sqrt{c}\omega_p}\right)\gamma_-\right]_\alpha^{\ \beta}.
\end{aligned} \tag{4.10}$$

We abbreviate the light-cone momenta p_-, q_- as p, q henceforth.

Vacuum energy density is given by

$$\begin{aligned}
\langle\Phi|\mathcal{H}|\Phi\rangle_{\mathrm{ren}} &\equiv \langle\Phi|\mathcal{H}|\Phi\rangle - \langle 0|\mathcal{H}|0\rangle = \int\frac{dp}{2\pi}p_+N\frac{2\Phi^2(p)}{1+\Phi^2(p)} \\
&+\int\frac{dpdq}{(2\pi)^2(p-q)^2}\frac{g^2(N^2-1)}{4\omega_q\omega_p}\frac{1}{1+\Phi^2(q)}\frac{1}{1+\Phi^2(p)} \\
&\left[\left(\omega_q\omega_p+qp+cm^2\right)(\Phi(q)-\Phi(p))^2+\left(\omega_q\omega_p-qp-cm^2\right)(1+\Phi(q)\Phi(p))^2\right. \\
&\left.+2\sqrt{c}m(q-p)\left(\Phi(q)-\Phi(p)\right)\left(1+\Phi(q)\Phi(p)\right)\right],
\end{aligned} \tag{4.11}$$

where the light-cone energy is given as

$$p_+ = \frac{-sp_- + \omega_p}{c}, \qquad \omega_p = \sqrt{(p_-)^2+cm^2}. \tag{4.12}$$

We have subtracted the vacuum energy of the Fock vacuum to obtain the renormalized vacuum energy $\langle\Phi|\mathcal{H}|\Phi\rangle_{\mathrm{ren}}$. The order parameter $\Phi(p_-)$ is determined by the extremum condition

$$\begin{aligned}
0 = \frac{\delta\langle\Phi|\mathcal{H}|\Phi\rangle_{\mathrm{ren}}}{\delta\Phi(p)} &= \frac{2N}{\pi(1+\Phi^2(p))^2}\left[p_+\Phi(p)\right. \\
&-\frac{g^2(N^2-1)}{8\pi N}\int\frac{dq}{(p-q)^2}\frac{1}{1+\Phi^2(q)}\frac{1}{\omega_q\omega_p} \\
&\left\{2(qp+cm^2)\left(1+\Phi(q)\Phi(p)\right)\left(\Phi(q)-\Phi(p)\right)\right. \\
&\left.\left.+\sqrt{c}m(q-p)\left((1+\Phi(q)\Phi(p))^2-(\Phi(q)-\Phi(p))^2\right)\right\}\right].
\end{aligned} \tag{4.13}$$

This equation is the gap equation in the gauge $A_- = 0$. The gap equation in the axial gauge $(c = 1, A_1 = 0)$ for massless QCD_2 is given before [17], [20]. The gap equation (4.13) in our gauge $A_- = 0$ depends on the parameter $c = -\cos\theta, s = \sin\theta$ defining the interpolating quantization

surface. Even in the case of $m=0$, the first term of our gap equation contains the factor p_+ which is asymmetric in $p_- \leftrightarrow -p_-$ as seen in eq.(4.12). Therefore it will give solutions $\Phi(p_-)$ asymmetric in $p_- \leftrightarrow -p_-$ which is different from the axial gauge solution. We hope that this gauge dependence should disappear when we compute gauge invariant quantities like the chiral condensate

$$\langle\Phi|\,\bar{\psi}(x)\psi(x)\,|\Phi\rangle = N\int_{-\infty}^{\infty}\frac{dp_-}{2\pi}\left[\frac{2\Phi(p_-)}{1+\Phi^2(p_-)}\frac{p_-}{\sqrt{c}\omega_p}-\frac{1-\Phi^2(p_-)}{1+\Phi^2(p_-)}\frac{m}{\omega_p}\right]. \tag{4.14}$$

A partial numerical evidence for this gauge independence has been given already [18]–[20].

It is an interesting problem to study two-dimensional QCD with matter in adjoint representation such as supersymmetric QCD [5]. We are looking for more powerful methods than 1/N expansion.

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Appendix

We summarize our notations and useful formulas. Our γ matrices in two dimensions are

$$\begin{aligned}
\gamma^0 &= \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \gamma^1 = i\sigma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \\
\gamma_5 &= \gamma^0\gamma^1 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\
\gamma^+ &= \gamma^0 \sin\frac{\theta}{2} + \gamma^1 \cos\frac{\theta}{2} = \begin{pmatrix} 0 & -i\sqrt{1-s} \\ i\sqrt{1+s} & 0 \end{pmatrix}, \\
\gamma^- &= \gamma^0 \sin\frac{\theta}{2} - \gamma^1 \cos\frac{\theta}{2} = \begin{pmatrix} 0 & -i\sqrt{1+s} \\ -i\sqrt{1-s} & 0 \end{pmatrix}.
\end{aligned} \tag{A.1}$$

The fermion ψ is a two component Dirac spinor

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}. \tag{A.2}$$

The Lagrangian of the free massive fermion and the conjugate momentum π for ψ are

$$\mathcal{L}_m = (\,\psi_R^* \quad \psi_L^*\,)\begin{pmatrix} i(\sqrt{1+s}\partial_+ - \sqrt{1-s}\partial_-) & im \\ -im & i(\sqrt{1-s}\partial_+ - \sqrt{1+s}\partial_-) \end{pmatrix}\begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}, \tag{A.3}$$

$$\pi(x) = \frac{\partial \mathcal{L}_\sigma}{\partial \partial_+ \psi(x)} = i\bar{\psi}\gamma^+ = (\, \psi_R^* \quad \psi_L^* \,) \begin{pmatrix} i\sqrt{1+s} & 0 \\ 0 & i\sqrt{1-s} \end{pmatrix}. \tag{A.4}$$

Spinors $u(p_-)$ and $v(p_-)$ with positive and negative energies are

$$u(p_-) = \frac{1}{\sqrt{2\omega_p}} \begin{pmatrix} \frac{1}{(1+s)^{\frac{1}{4}}}(\omega_p + p_-)^{\frac{1}{2}} \\ \frac{i}{(1-s)^{\frac{1}{4}}}(\omega_p - p_-)^{\frac{1}{2}} \end{pmatrix}, \quad v(p_-) = \frac{1}{\sqrt{2\omega_p}} \begin{pmatrix} \frac{1}{(1+s)^{\frac{1}{4}}}(\omega_p + p_-)^{\frac{1}{2}} \\ \frac{-i}{(1-s)^{\frac{1}{4}}}(\omega_p - p_-)^{\frac{1}{2}} \end{pmatrix}. \tag{A.5}$$

These spinors satisfy equations of motion and completeness relations

$$(\gamma^\mu p_\mu - m)u(p) = 0, \qquad (\gamma^\mu p_\mu + m)v(p) = 0, \tag{A.6}$$

$$u(p_-)\bar{u}(p_-) = \frac{1}{2\omega_p}\left[\gamma^\mu p_\mu + m\right], \quad v(p_-)\bar{v}(p_-) = \frac{1}{2\omega_p}\left[\gamma^\mu p_\mu - m\right]. \tag{A.7}$$

The Bogoliubov transformation is an orthogonal transformation between annihilation operator of quark with momentum p_- and the creation operator of antiquark with momentum $-p_-$ as given in eq.(4.8). By defining a new spinors U and V, we can rewrite the fermion field in the first line to the second line of eq.(4.9)

$$\begin{aligned} U(p_-) &= \frac{1}{\sqrt{1+\Phi^2(p_-)}}\left[u(p_-) + \Phi(p_-)v(-p_-)\right], \\ V(-p_-) &= \frac{1}{\sqrt{1+\Phi^2(p_-)}}\left[v(-p_-) - \Phi(p_-)u(p_-)\right]. \end{aligned} \tag{A.8}$$

References

[1] P.A.M. Dirac, *Rev. Mod. Phys.* **21** (1949) 392; S. Weinberg, *Phys. Rev.* **150** (1966) 1313.

[2] G. 't Hooft, *Nucl. Phys.* **B75** (1975) 461.

[3] H-C. Pauli and S. Brodsky, *Phys. Rev.* **D32** (1985) 1993, ibid 2001; K. Hornbostel, Ph.D Thesis, SLAC report 333 (1988).

[4] K. Hornbostel, S.J. Brodsky, and H-C. Pauli, *Phys. Rev.* **D41** (1990) 3814; R.J. Perry, A. Harindranath and K.G. Wilson, *Phys. Rev. Lett.* **65** (1990) 2959; K.G. Wilson et.al., *Phys. Rev.* **D49** (1994) 6720; K. Harada, T. Sugihara, M. Taniguchi, and M. Yahiro, *Phys. Rev.* **D49** (1994) 4226 ; T. Sugiura, M. Matsuzaki, and M. Yahiro, *Phys. Rev.* **D50** (1994) 5274.

[5] Y. Matsumura, N. Sakai, and T. Sakai, *Phys. Rev.* **D52** (1995) 2446.

[6] T. Maskawa and K. Yamawaki, *Prog. Theor. Phys.* **56** (1976) 270; N. Nakanishi and K. Yamawaki, *Nucl. Phys.* **B122** (1977) 15.

[7] C. Bender, S. Pinsky and B. van de Sanda, *Phys. Rev.* **D48** (1993) 816, *Phys. Rev.* **D49** (1994) 2001, *Phys. Rev.* **D51** (1995) 726; A.C. Kalloniatis, H-C. Pauli, and S. Pinsky, *Phys. Rev.* **D50** (1994) 6633; Yoonbai Kim, S. Tsujimaru and K. Yamawaki, *Phys. Rev. Lett.* **74** (1995) 4771; I. Pesando, *Mod. Phys. Lett.* **A10** (1995) 525, hep-th/9506100.

[8] J.E. Hetrick and Y. Hosotani, *Phys. Lett.* **230** (1989) 88; A.V. Smilga, *Phys. Rev.* **D49** (1994) 6836; A.C. Kalloniatis and D.G. Robertson, *Phys. Rev.* **D50** (1994) 5262; M. Maeno, *Phys. Lett.* **B320** (1994) 83; F. Lenz, M. Shifman and M.Thies, *Phys. Rev.* **D51** (1995) 7060; M. Tachibana, hep-th.9504026; A.C. Kalloniatis, hep-th.9509027; S.S. Pinsky and A.C. Kalloniatis, hep-th.9509027.

[9] F. Lenz, M. Thies, S. Levit and K. Yazaki, *Ann. Phys.* **208** (1991) 1.

[10] K. Hornbostel, *Phys. Rev.* **D45** (1992) 3781.

[11] S. Coleman and E. Weinberg, *Phys. Rev.* **D7** (1973) 1888; J. M. Cornwall, R. Jackiw, and E. Tomboulis, *Phys. Rev.* **D10** (1974) 2428.

[12] D. J. Gross and A. Neveu, *Phys. Rev.* **D10** (1974) 3235.

[13] S. Coleman, R. Jackiw and H.D. Politzer, *Phys. Rev.* **D10** (1974) 2491.

[14] L. F. Abbott, J. S. Kang and H. J. Schnitzer, *Phys. Rev.* **D13** (1976) 2212.

[15] G. 't Hooft, *Nucl. Phys.* **B72** (1974) 461.

[16] C.G. Callan, N. Coote and D.J. Gross, *Phys. Rev.* **D13** (1976) 1649; M.B. Einhorn, *Phys. Rev.* **D14** (1976) 3451.

[17] I. Bars and M. Green, *Phys. Rev.* **D17** (1978) 537.

[18] A. P. Zhitnitsky, *Phys. Lett.* **B165** (1985) 405.

[19] M. Burkhardt, hep-ph.9409333

[20] M. Li, *Phys. Rev.* **D34** (1986) 3888.

[21] E. Witten, *Nucl. Phys.* **B145** (1978) 110.

[22] V. L. Berezinski, *Sov. Phys. JETP.* **32** (1971) 493; J. M. Kosterlitz and D. J. Thouless, *J. Phys.* **C6** (1973) 1181.

[23] S. Coleman, *Commun. Math. Phys.* **31** (1973) 259.

[24] M. Thies and K. Ohta, *Phys. Rev.* **D48** (1993) 5883.

[25] L. Adler and A. C. Davis, *Nucl. Phys.* **B244** (1984) 469.

Figure Caption

Fig. 1 Dispersion relations of a free massive particle on (a) usual, (b) an interpolating and (c) light-cone quantization surfaces.